Example 7-1 Calculation of Heat-Exchanger Size from Known Temperatures

Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/kg · °C. The fluids are used in a counterflow double-pipe heat exchanger, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/m^2 · °C. Calculate the heat-exchanger area.

Solution

The total heat transfer is determined from the energy absorbed by the water:

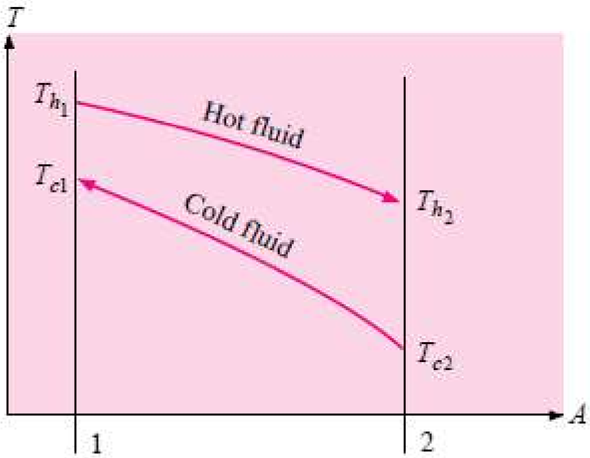
$$q = \dot{m}_w c_w \Delta T_w = (68)(4180)(75 - 35) = 11.37 \text{ MJ/min} \qquad [a]$$
$$= 189.5 \text{ kW} \qquad [6.47 \times 10^5 \text{ Btu/h}]$$

Since all the fluid temperatures are known, the LMTD can be calculated by using the temperature scheme in Figure 7-8b

$$\Delta T_m = \frac{(110 - 75) - (75 - 35)}{\ln[(110 - 75)/(75 - 35)]} = 37.44°\text{C} \qquad [b]$$

Then, since $q = UA\,\Delta T_m$,

$$A = \frac{1.895 \times 10^5}{(320)(37.44)} = 15.82 \text{ m}^2 \qquad [170 \text{ ft}^2]$$



(b)

Shell-and-Tube Heat Exchanger

Example 7-2

Instead of the double-pipe heat exchanger of Example 10-4, it is desired to use a shell-and-tube exchanger with the water making one shell pass and the [Example 7-1] ıbe passes. Calculate the area required for this exchanger, assuming that the ove[...] coefficient remains at 320 W/m^2 · °C.

Solution

To solve this problem, we determine a correction factor from Figure 10-8 to be used with the LMTD calculated on the basis of a counterflow exchanger. The parameters according to the nomenclature of Figure 10-8 are

$$T_1 = 35°\text{C} \qquad T_2 = 75°\text{C} \qquad t_1 = 110°\text{C} \qquad t_2 = 75°\text{C}$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{75 - 110}{35 - 110} = 0.467$$

Figure 7-9

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{35 - 75}{75 - 110} = 1.143$$

so the correction factor is

$$F = 0.81$$

and the heat transfer is

$$q = UAF\,\Delta T_m$$

so that

$$A = \frac{1.895 \times 10^5}{(320)(0.81)(37.44)} = 19.53\ \text{m}^2 \quad [210\ \text{ft}^2]$$

Design of Shell-and-Tube Heat Exchanger

Example 7-3

Water at the rate of 30,000 lb$_m$/h [3.783 kg/s] is heated from 100 to 130°F [37.78 to 54.44°C] in a shell-and-tube heat exchanger. On the shell side one pass is used with water as the heating fluid, 15,000 lb$_m$/h [1.892 kg/s], entering the exchanger at 200°F [93.33°C]. The overall heat-transfer coefficient is 250 Btu/h · ft^2 · °F [1419 W/m^2 · °C], and the average water velocity in the $\frac{3}{4}$-in [1.905-cm] diameter tubes is 1.2 ft/s [0.366 m/s]. Because of space limitations, the tube length must not be longer than 8 ft [2.438 m]. Calculate the number of tube passes, the number of tubes per pass, and the length of the tubes, consistent with this restriction.

Solution

We first assume one tube pass and check to see if it satisfies the conditions of this problem. The exit temperature of the hot water is calculated from

$$q = \dot{m}_c c_c\,\Delta T_c = \dot{m}_h c_h\,\Delta T_h$$

$$\Delta T_h = \frac{(30{,}000)(1)(130 - 100)}{(15{,}000)(1)} = 60°\text{F} = 33.33°\text{C} \qquad [a]$$

$$\Delta T_h = \frac{\dot{m}_c c_c \Delta T_c}{\dot{m}_h c_h} = \frac{(3.783)(54.44 - 37.78)}{(1.892)} = 33.33\ °\text{C}$$

so

$$T_{h,\text{exit}} = 93.33 - 33.33 = 60°\text{C}$$

The total required heat transfer is obtained from Equation (*a*) for the cold fluid:

$$q = (3.783)(4182)(54.44 - 37.78) = 263.6 \text{ kW} \quad [8.08 \times 10^5 \text{ Btu/h}]$$

For a counterflow exchanger, with the required temperature

$$\text{LMTD} = \Delta T_m = \frac{(93.33 - 54.44) - (60 - 37.78)}{\ln[(93.33 - 54.44)/(60 - 37.78)]} = 29.78°\text{C}$$

$$q = UA\,\Delta T_m$$

$$A = \frac{2.636 \times 10^5}{(1419)(29.78)} = 6.238 \text{ m}^2 \quad [67.1 \text{ ft}^2] \qquad [b]$$

Using the average water velocity in the tubes and the flow rate, we calculate the total flow area with

$$\dot{m}_c = \rho A u$$

$$A = \frac{3.783}{(1000)(0.366)} = 0.01034 \text{ m}^2 \qquad [c]$$

This area is the product of the number of tubes and the flow area per tube:

$$0.01034 = n\frac{\pi d^2}{4}$$

$$n = \frac{(0.01034)(4)}{\pi(0.01905)^2} = 36.3$$

or $n = 36$ tubes. The surface area per tube per meter of length is

$$\pi d = \pi(0.01905) = 0.0598 \text{ m}^2/\text{tube}\cdot\text{m}$$

We recall that the total surface area required for a one-tube-pass exchanger was calculated in Equation (*b*) as 6.238 m^2. We may thus compute the length of tube for this type of exchanger from

$$n\pi d\,L = 6.238$$

$$L = \frac{6.238}{(36)(0.0598)} = 2.898 \text{ m}$$

This length is greater than the allowable 2.438 m, so we must use more than one tube pass. When we increase the number of passes, we correspondingly increase the total surface area required because of the reduction in LMTD caused by the correction factor F. We next try two tube passes.

From Figure 7-9 F=0.88, and thus

$$A_{\text{total}} = \frac{q}{UF\Delta T_m} = \frac{2.636 \times 10^5}{(1419)(0.88)(29.78)} = 7.089 \text{ m}^2$$

The number of tubes per pass is still 36 because of the velocity requirement. For the two-tube-pass exchanger the total surface area is now related to the length by

$$A_{\text{total}} = 2n\pi d\,L$$

so that

$$L = \frac{7.089}{(2)(36)(0.0598)} = 1.646 \text{ m} \quad [5.4 \text{ ft}]$$

This length is within the 2.438-m requirement, so the final design choice is

Number of tubes per pass = 36

Number of passes = 2

Length of tube per pass = 1.646 m [5.4 ft]

Cross-Flow Exchanger with One Fluid Mixed — Example 7-4

A heat exchanger like that shown below ; used to heat an oil in the tubes ($c = 1.9$ kJ/kg · °C) from 15°C to 85°C. Blowing across the outside of the tubes is steam that enters at 130°C and leaves at 110°C with a mass flow of 5.2 kg/sec. The overall heat-transfer coefficient is 275 W/m^2 · °C and c for steam is 1.86 kJ/kg · °C. Calculate the surface area of the heat exchanger.

Solution

The total heat transfer may be obtained from an energy balance on the steam

$$q = \dot{m}_s c_s \,\Delta T_s = (5.2)(1.86)(130 - 110) = 193 \text{ kW}$$

We can solve for the area from Equation (10-13). The value of ΔT_m is calculated *as if the exchanger were counterflow double pipe* (i.e., as shown in Figure Example 10-7). Thus,

$$\Delta T_m = \frac{(130 - 85) - (110 - 15)}{\ln\left(\frac{130 - 85}{110 - 15}\right)} = 66.9°\text{C}$$

$$\Delta T_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left[\frac{(T_{h1} - T_{c1})}{(T_{h2} - T_{c2})}\right]}$$

$$T_1 = 130 \qquad T_2 = 110 \qquad t_1 = 15 \qquad t_2 = 85°\text{C}$$

and we calculate

$$R = \frac{130 - 110}{85 - 15} = 0.286 \qquad P = \frac{85 - 15}{130 - 15} = 0.609$$

Consulting Figure 7-11 we find

$$F = 0.97$$

so the area is calculated from

$$A = \frac{q}{UF\,\Delta T_m} = \frac{193{,}000}{(275)(0.97)(66.9)} = 10.82 \text{ m}^2$$

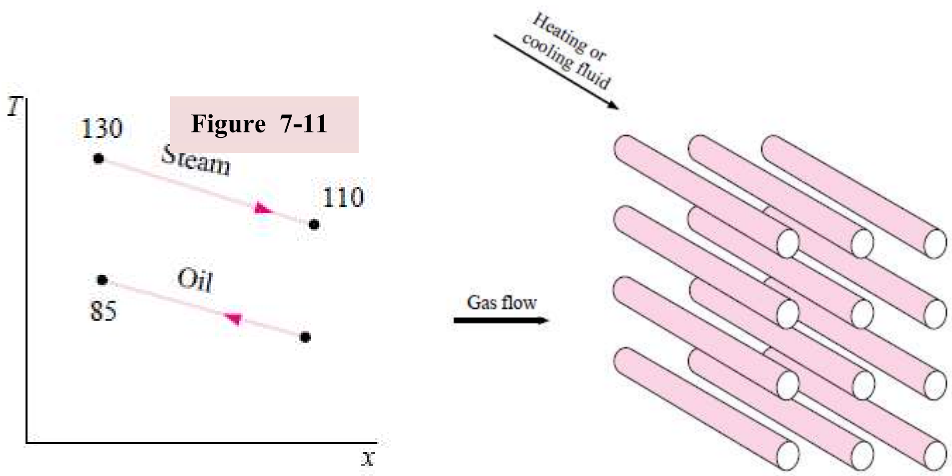


Figure 7-11

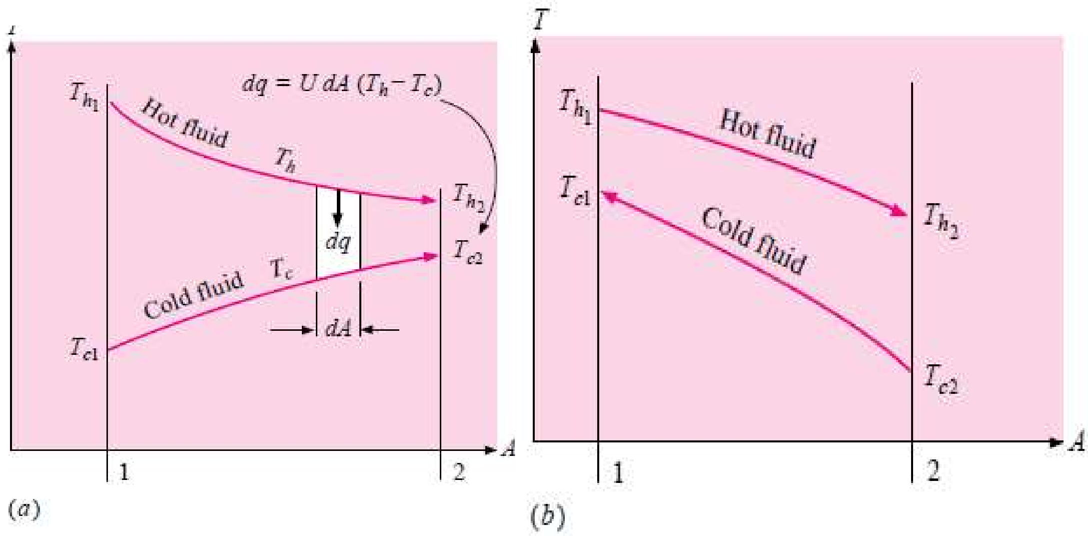
7-6 Effectiveness-NTU Method

The LMTD approach to heat-exchanger analysis is useful when the inlet and outlet temperatures are known or are easily determined. The LMTD is then easily calculated, and the heat flow, surface area, or overall heat-transfer coefficient may be determined. When the inlet or exit temperatures are to be evaluated for a given heat exchanger, the analysis frequently involves an iterative procedure because of the logarithmic function in the LMTD. In these cases the analysis is performed more easily by utilizing a method based on the effectiveness of the heat exchanger in transferring a given amount of heat. The effectiveness method also offers many advantages for analysis of problems in which a comparison between various types of heat exchangers must be made for purposes of selecting the type best suited to accomplish a particular heat-transfer objective.

We define the heat-exchanger effectiveness as

$$\text{Effectiveness} = \epsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid. Consider the parallel-flow and counterflow heat exchangers shown in Figure 7-8.



For the parallel-flow exchanger

$$q = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1}) \qquad 7\text{-}14$$

For the counterflow exchanger

$$q = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c1} - T_{c2}) \qquad 7\text{-}15$$

Maximum possible heat transfer is expressed as

$$q_{max} = (\dot{m}c)_{min}\left(T_{h_{inlet}} - T_{c_{inlet}}\right) \qquad 7\text{-}16$$

The minimum fluid may be either the hot or cold fluid, depending on the mass-flow rates and specific heats.

For the parallel-flow exchanger

$$\epsilon_h = \frac{\dot{m}_h c_h (T_{h1} - T_{h2})}{\dot{m}_h c_h (T_{h1} - T_{c1})} = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c1})} \qquad 7\text{-}17$$

$$\epsilon_c = \frac{\dot{m}_c c_c (T_{c2} - T_{c1})}{\dot{m}_c c_c (T_{h1} - T_{c1})} = \frac{(T_{c2} - T_{c1})}{(T_{h1} - T_{c1})} \qquad 7\text{-}18$$

The subscripts on the effectiveness symbols designate the fluid that has the minimum value of$\dot{m}c$.

For the counterflow exchanger:

$$\epsilon_h = \frac{\dot{m}_h c_h (T_{h1} - T_{h2})}{\dot{m}_h c_h (T_{h1} - T_{c2})} = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c2})} \qquad 7\text{-}19$$

$$\epsilon_c = \frac{\dot{m}_c c_c (T_{c1} - T_{c2})}{\dot{m}_c c_c (T_{h1} - T_{c2})} = \frac{(T_{c1} - T_{c2})}{(T_{h1} - T_{c2})} \qquad 7\text{-}20$$

In a general way the effectiveness is expressed as

$$\epsilon = \frac{\Delta T(minimum\ fluid)}{Maximum\ temperature\ difference\ in\ heat\ exchanger} \qquad 7\text{-}21$$

The minimum fluid is always the one experiencing the larger temperature difference in the heat exchanger, and the maximum temperature difference in the heat exchanger is always the difference in inlet temperatures of the hot and cold fluids.

We may derive an expression for the effectiveness in parallel flow double-pipe as follows. Rewriting Equation (7-10), we have

$$\ln\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = -UA\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) \qquad 7\text{-}10$$

$$\ln\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = -UA\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) = \frac{-UA}{\dot{m}_c c_c}\left(1 + \frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right) \qquad 7\text{-}22$$

Or

$$\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}} = \exp\left[\frac{-UA}{\dot{m}_c c_c}\left(1+\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)\right] \qquad 7\text{-}23$$

If the cold fluid is the minimum fluid,

$$\epsilon = \frac{(T_{c2}-T_{c1})}{(T_{h1}-T_{c1})}$$

$$\dot{m}_h c_h(T_{h1}-T_{h2}) = \dot{m}_c c_c(T_{c2}-T_{c1})$$

$$T_{h2} = T_{h1} + \frac{\dot{m}_c c_c}{\dot{m}_h c_h}(T_{c1}-T_{c2})$$

Rewriting the temperature ratio in Equation in the left side (7-23) gives

$$\frac{T_{h2}-T_{c2}}{T_{h1}-T_{c1}} = \frac{T_{h1}+\left(\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)(T_{c1}-T_{c2})-T_{c2}+(T_{c1}-T_{c1})}{T_{h1}-T_{c1}} \qquad 7\text{-}24$$

Equation (7-24) may now be rewritten

$$\frac{(T_{h1}-T_{c1})+\left(\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)(T_{c1}-T_{c2})+(T_{c1}-T_{c2})}{(T_{h1}-T_{c1})} = 1-\left(1+\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)\epsilon$$

Inserting this relation back in Equation (7-23) gives for the effectiveness

$$\epsilon = \frac{1-\exp\left[\frac{-UA}{\dot{m}_c c_c}\left(1+\frac{\dot{m}_c c_c}{\dot{m}_h c_h}\right)\right]}{1+\frac{\dot{m}_c c_c}{\dot{m}_h c_h}} \qquad 7\text{-}25$$

It may be shown that the same expression results for the effectiveness when the hot fluid is the minimum fluid, except that $\dot{m}_c c_c$ and $\dot{m}_h c_h$ are interchanged. As a consequence, the effectiveness is usually written

$$\epsilon = \frac{1-\exp\left[\frac{-UA}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right)\right]}{1+\frac{C_{min}}{C_{max}}} \qquad \text{parallel flow} \qquad 7\text{-}26$$

Where

$C = \dot{m}c$ is define as the capacity rate.

A similar analysis may be applied to the counterflow case, and the following relation for effectiveness results:

$$\epsilon = \frac{1-\exp\left[\frac{-UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]}{1-\left(\frac{C_{min}}{C_{max}}\right)\exp\left[\frac{-UA}{C_{min}}\left(1-\frac{C_{min}}{C_{max}}\right)\right]} \qquad \text{counter flow} \qquad 7\text{-}27$$

The grouping of terms $\frac{UA}{C_{min}}$ is called the *number of transfer units* (NTU) since it is indicative of the size of the heat exchanger.

Effectiveness and Heat-Transfer Rates

We must caution the reader about misinterpreting the physical meaning of heat-exchanger effectiveness. Just because a heat exchanger has a high *effectiveness* at a certain flow condition does not mean that it will have a higher heat-transfer *rate* than at some low effectiveness condition. High values of ε correspond to small temperature differences between the hot and cold fluid, while higher heat-transfer *rates* result from larger temperature differences (i.e., a greater driving potential). In a thermodynamic sense, higher effectiveness values correspond to reduced values of thermodynamic irreversibility and smaller entropy generation. To achieve *both* high heat transfer *and* high effectiveness one must increase the value of the UA product, either by increasing the size (and cost) of the exchanger or by forcing the fluid(s) through the heat exchanger at higher velocities to produce increased convection coefficients. Or, one may employ so-called heat-transfer augmentation techniques to increase the value of UA.

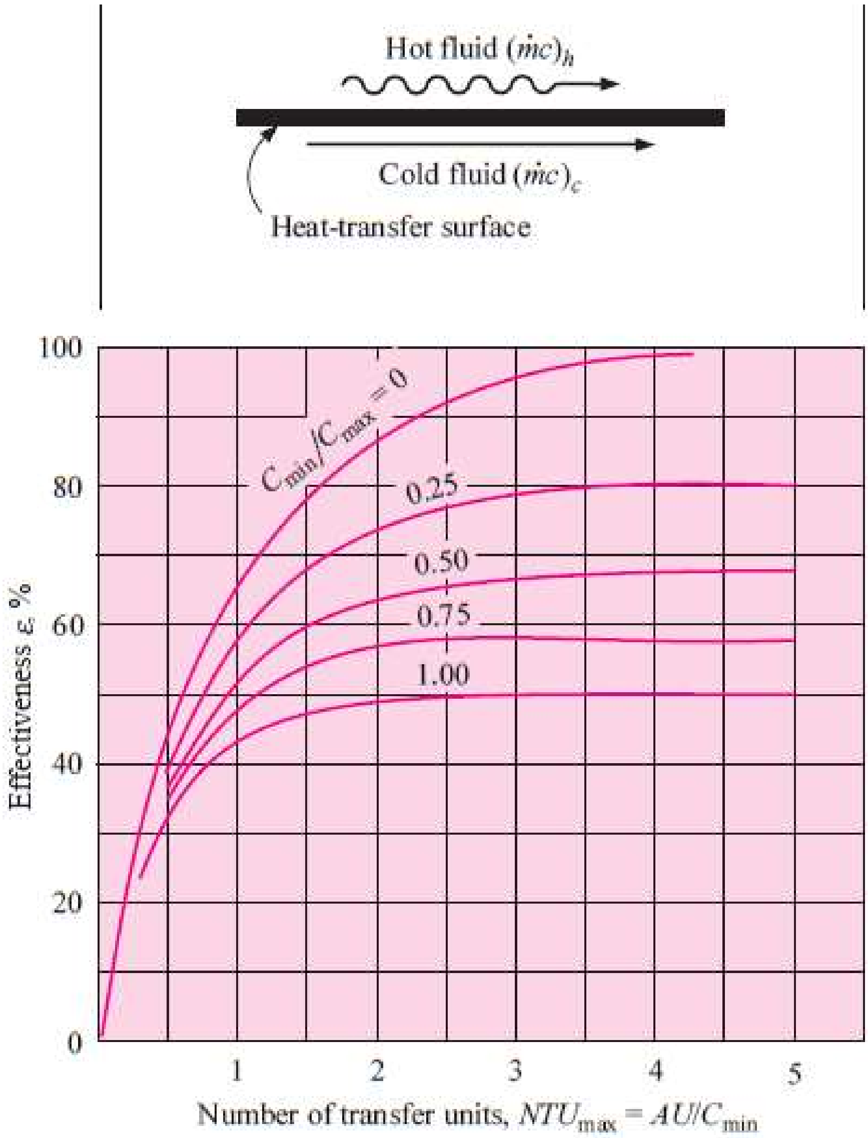


Figure 7-13 Effectiveness for parallel-flow exchanger performance.

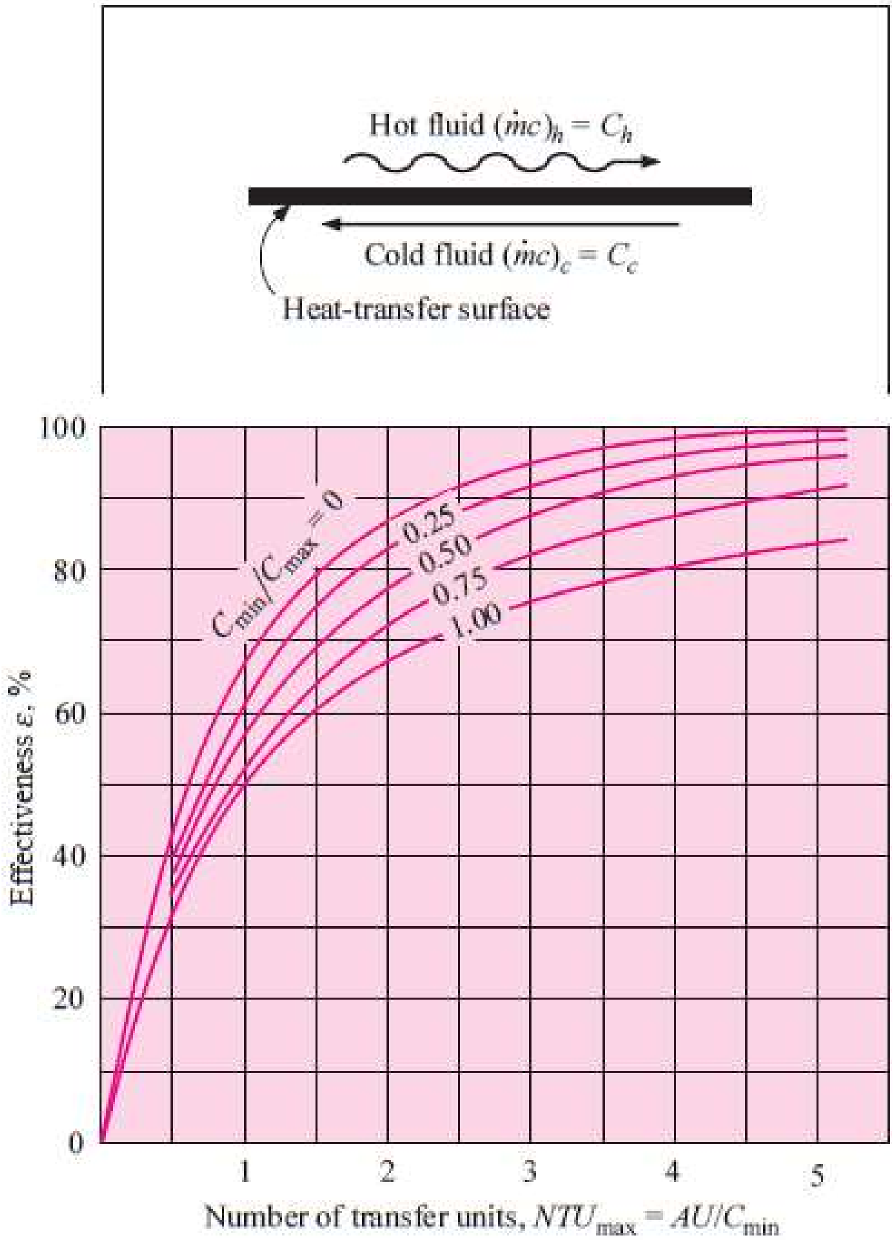


Figure 7-14 Effectiveness for counter flow exchanger performance.

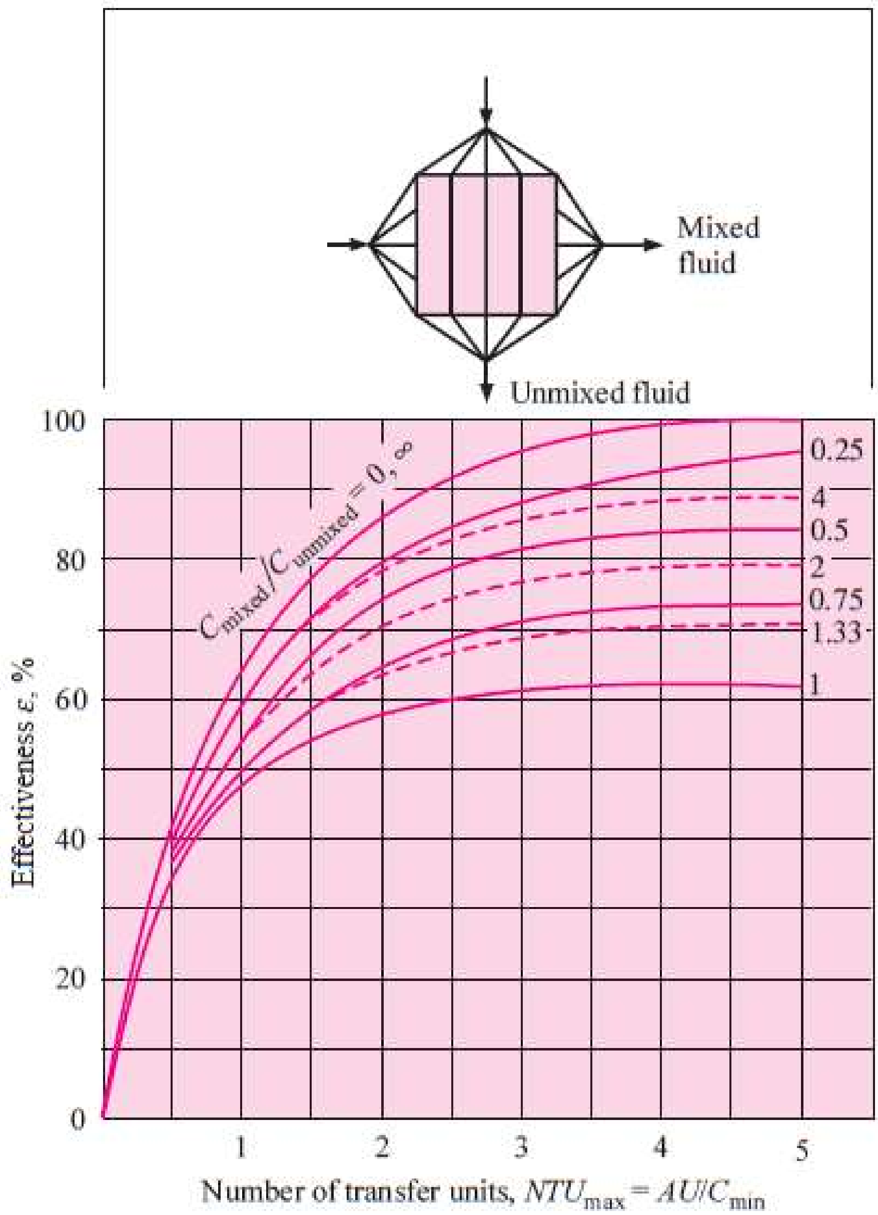


Figure 7-15 Effectiveness for cross-flow exchanger with one fluid mixed.

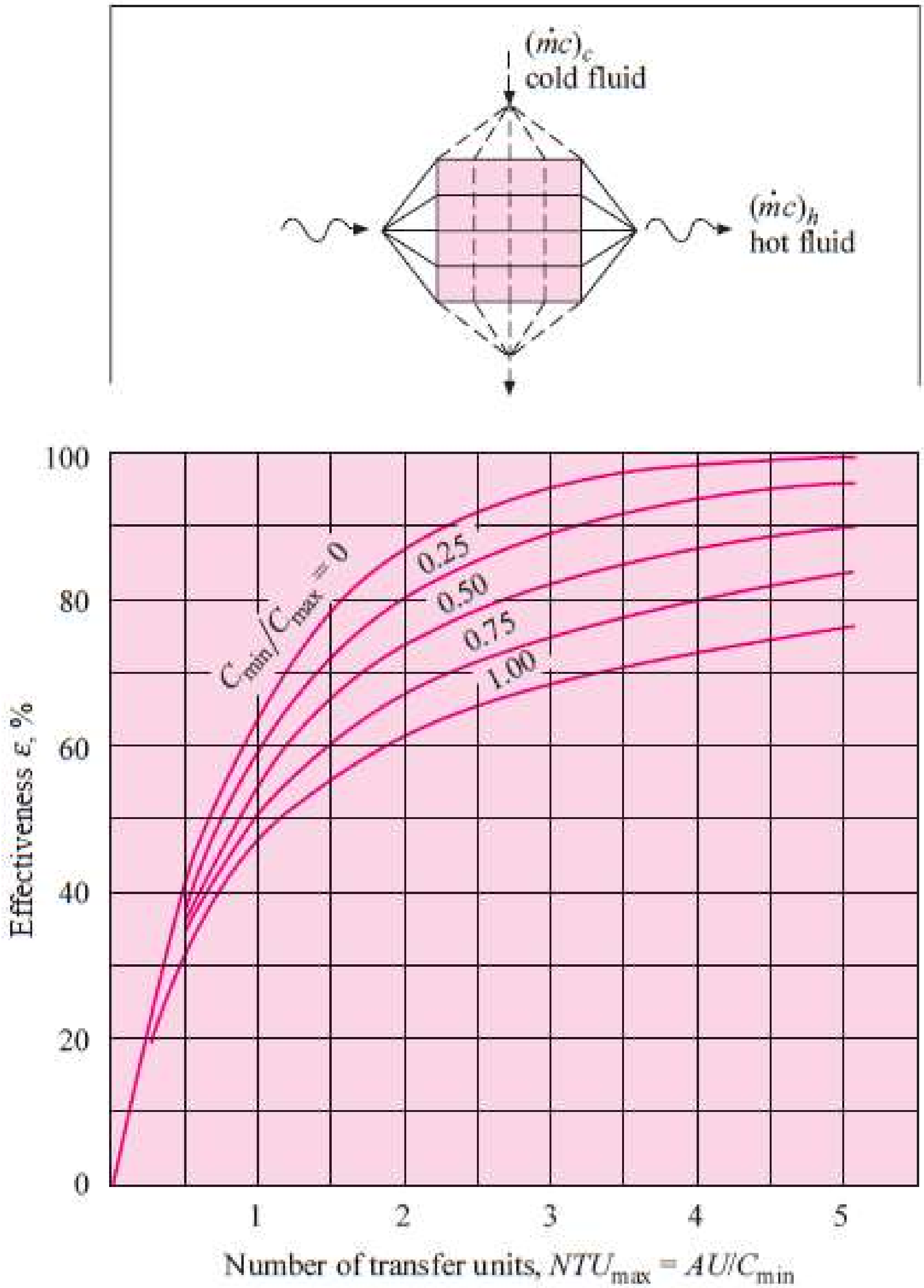


Figure 7-16 Effectiveness for cross-flow exchanger with fluids unmixed.

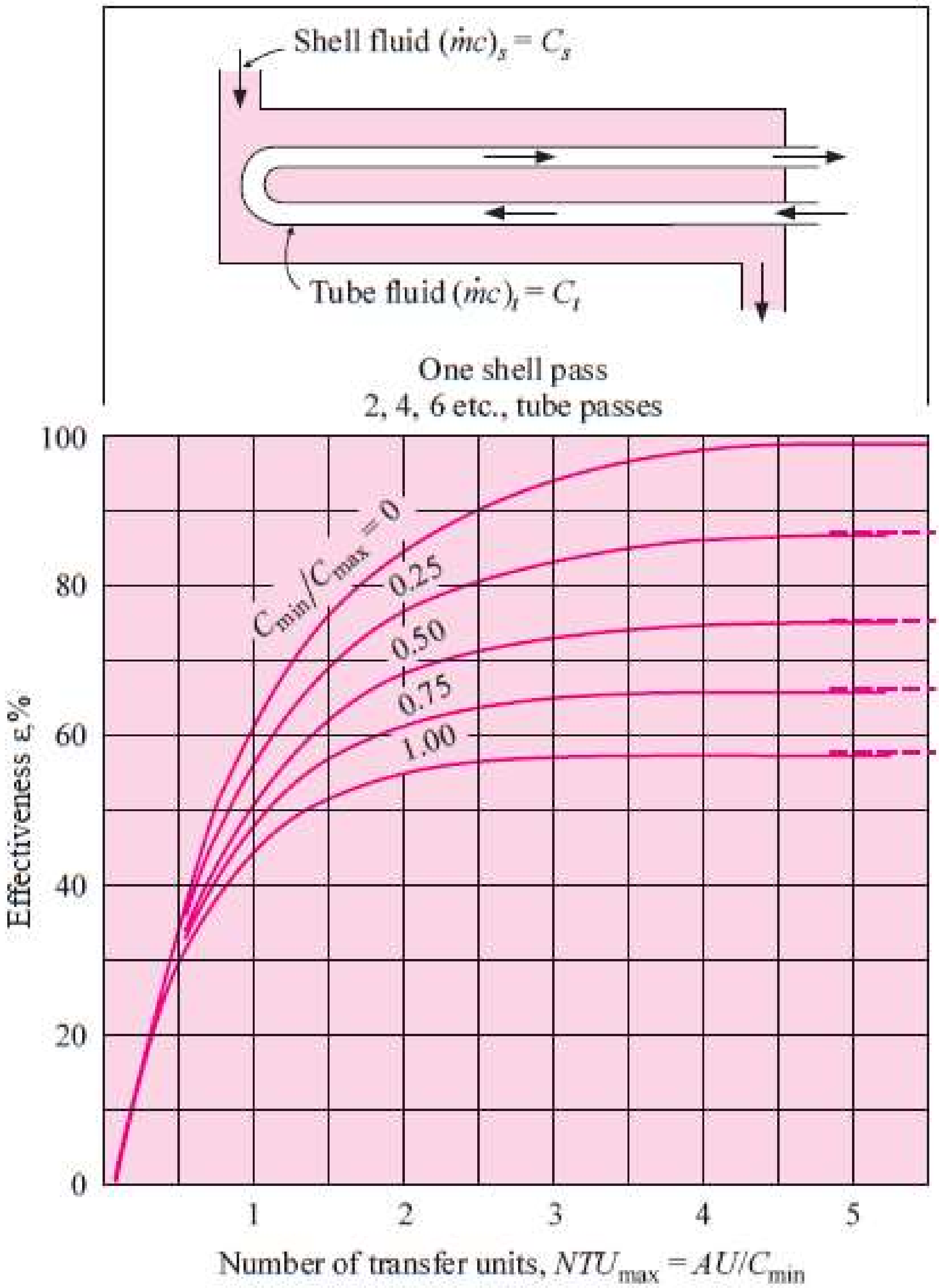


Figure 7-17 Effectiveness for 1-2 parallel counterflow exchanger performance.

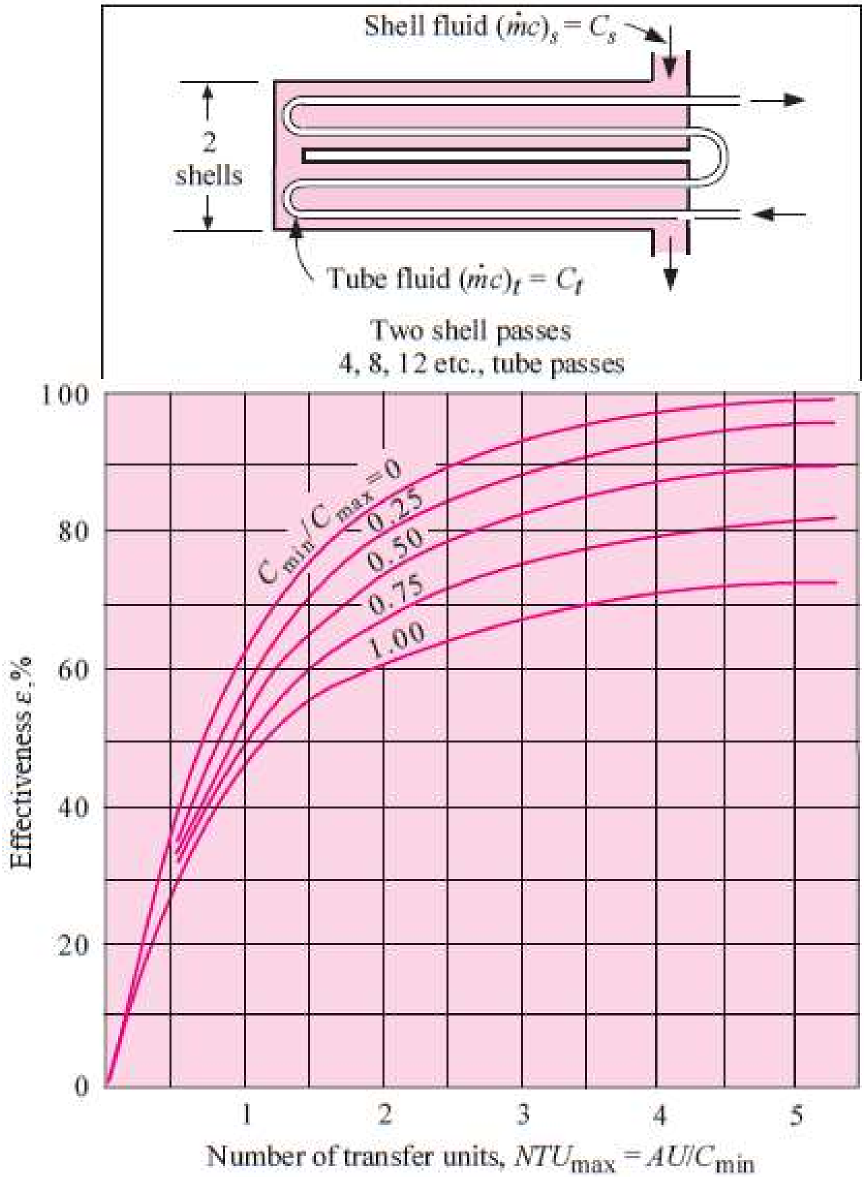


Figure 7-18 Effectiveness for 2-4 multi-pass counter-flow exchanger performance.

Table 7-1 Heat-exchanger effectiveness relations.

$N = \text{NTU} = \dfrac{UA}{C_{min}}$ $\qquad C = \dfrac{C_{min}}{C_{max}}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$\epsilon = \dfrac{1-\exp[-N(1+C)]}{1+C}$
Counterflow	$\epsilon = \dfrac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
Counterflow, $C = 1$	$\epsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\epsilon = 1-\exp\left[\dfrac{\exp(-NCn)-1}{Cn}\right]$ where $n = N^{-0.22}$
Both fluids mixed	$\epsilon = \left[\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-NC)} - \dfrac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unmixed	$\epsilon = (1/C)\{1-\exp[-C(1-e^{-N})]\}$
C_{max} unmixed, C_{min} mixed	$\epsilon = 1-\exp\{-(1/C)[1-\exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\epsilon = 2\left\{1+C+(1+C^2)^{1/2} \times \dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1-\exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
Multiple shell passes, $2n$, $4n$, $6n$ tube passes ($\epsilon_p =$ effectiveness of each shell pass, $n =$ number of shell passes)	$\epsilon = \dfrac{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - 1}{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - C}$
Special case for $C = 1$	$\epsilon = \dfrac{n\epsilon_p}{1+(n-1)\epsilon_p}$
All exchangers with $C = 0$	$\epsilon = 1 - e^{-N}$